OCR
RECOGNISING ACHIEVEMENT

ADVANCED GCE
MATHEMATICS
Probability & Statistics 4

4735

Candidates answer on the Answer Booklet

OCR Supplied Materials:
- 8 page Answer Booklet
- List of Formulae (MF1)

Other Materials Required:
- Scientific or graphical calculator

Thursday 24 June 2010
Morning

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

- Write your name clearly in capital letters, your Centre Number and Candidate Number in the spaces provided on the Answer Booklet.
- Use black ink. Pencil may be used for graphs and diagrams only.
- Read each question carefully and make sure that you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.
- You are permitted to use a graphical calculator in this paper.

INFORMATION FOR CANDIDATES

- The number of marks is given in brackets **[ ]** at the end of each question or part question.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- This document consists of **4** pages. Any blank pages are indicated.

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Turn over

1 For the variables A and B, it is given that $\text{Var}(A) = 9$, $\text{Var}(B) = 6$ and $\text{Var}(2A - 3B) = 18$.

(i) Find $\text{Cov}(A, B)$. **[3]**

(ii) State with a reason whether A and B are independent. **[1]**

2 The probability generating function of the discrete random variable X is $\dfrac{e^{4t^2}}{e^4}$. Find

(i) $\text{E}(X)$, **[3]**

(ii) $\text{P}(X = 2)$. **[3]**

3 X_1 and X_2 are continuous random variables. Random samples of 5 observations of X_1 and 6 observations of X_2 are taken. No two observations are equal. The 11 observations are ranked, lowest first, and the sum of the ranks of the observations of X_1 is denoted by R.

(i) Assuming that all rankings are equally likely, show that $\text{P}(R \leqslant 17) = \frac{2}{231}$. **[5]**

The marks of 5 randomly chosen students from School A and 6 randomly chosen students from School B, who took the same examination, achieving different marks, were ranked. The rankings are shown in the table.

Rank	1	2	3	4	5	6	7	8	9	10	11
School	A	A	A	B	A	A	B	B	B	B	B

(ii) For a Wilcoxon rank-sum test, obtain the exact smallest significance level for which there is evidence of a difference in performance at the two schools. **[2]**

4 The moment generating function of a continuous random variable Y, which has a χ^2 distribution with n degrees of freedom, is $(1 - 2t)^{-\frac{1}{2}n}$, where $0 \leqslant t < \frac{1}{2}$.

(i) Find $\text{E}(Y)$ and $\text{Var}(Y)$. **[5]**

For the case $n = 1$, the sum of 60 independent observations of Y is denoted by S.

(ii) Write down the moment generating function of S and hence identify the distribution of S. **[2]**

(iii) Use a normal approximation to estimate $\text{P}(S \geqslant 70)$. **[3]**

5 In order to test whether the median salary of employees in a certain industry who had worked for three years was £19 500, the salaries x, in thousands of pounds, of 50 randomly chosen employees were obtained.

(i) The values $|x - 19.5|$ were calculated and ranked. No two values of x were identical and none was equal to 19.5. The sum of the ranks corresponding to positive values of $(x - 19.5)$ was 867. Stating a required assumption, carry out a suitable test at the 5% significance level. **[10]**

(ii) If the assumption you stated in part **(i)** does not hold, what test could have been used? **[1]**

6 Nuts and raisins occur in randomly chosen squares of a particular brand of chocolate. The numbers of nuts and raisins are denoted by N and R respectively and the joint probability distribution of N and R is given by

$$\mathrm{f}(n, r) = \begin{cases} c(n+2r) & n = 0, 1, 2 \text{ and } r = 0, 1, 2, \\ 0 & \text{otherwise,} \end{cases}$$

where c is a constant.

(i) Find the value of c. **[3]**

(ii) Find the probability that there is exactly one nut in a randomly chosen square. **[2]**

(iii) Find the probability that the total number of nuts and raisins in a randomly chosen square is more than 2. **[2]**

(iv) For squares in which there are 2 raisins, find the mean number of nuts. **[4]**

(v) Determine whether N and R are independent. **[2]**

7 The continuous random variable X has probability density function given by

$$\mathrm{f}(x) = \begin{cases} \dfrac{x}{2\theta^2} & 0 \leqslant x \leqslant 2\theta, \\ 0 & \text{otherwise,} \end{cases}$$

where θ is an unknown positive constant.

(i) Find $\mathrm{E}(X^n)$, where $n \neq -2$, and hence write down the value of $\mathrm{E}(X)$. **[3]**

(ii) Find

(a) $\mathrm{Var}(X)$,

(b) $\mathrm{Var}(X^2)$.

[4]

(iii) Find $\mathrm{E}(X_1 + X_2 + X_3)$ and $\mathrm{E}(X_1^2 + X_2^2 + X_3^2)$, where X_1, X_2 and X_3 are independent observations of X. Hence construct unbiased estimators, T_1 and T_2, of θ and $\mathrm{Var}(X)$ respectively, which are based on X_1, X_2 and X_3. **[6]**

(iv) Find $\mathrm{Var}(T_2)$. **[2]**

8 For the events L and M, $\mathrm{P}(L \mid M) = 0.2$, $\mathrm{P}(M \mid L) = 0.4$ and $\mathrm{P}(M) = 0.6$.

(i) Find $\mathrm{P}(L)$ and $\mathrm{P}(L' \cup M')$. **[3]**

(ii) Given that, for the event N, $\mathrm{P}(N \mid (L \cap M)) = 0.3$, find $\mathrm{P}(L' \cup M' \cup N')$. **[3]**